Syllabus of Ph.D. Entrance Examination

15. MATHEMATICS

1. **Real Analysis:** Riemann integrate functions; improper integrate, their convergence and uniform convergence. Eulidean space R^{-}, Boizano – Weleratrass theorem, compact. Subsets of R•, Heine – Borel theorem, Fourier series. Continuity of functions on R", Differentiability of F: R• > Rm, Properties of differential, partial and directional derivatives, continuously differentiable functions. Taylor's series. Inverse function theorem, implicit function theorem. Integral functions, line and surface integrals, Green's theorem. Stoke's theorem.

2. **Complex Analysis:** Cauchy's theorem for convex regions, Power series representation of Analytic functions. Liouville's theorem, Fundamental theorem of algebra, Riemann's theorem on removable singularities, maximum modulus principle. Schwarz lemma, Open Mapping theorem, Casoratti–Weierstrass–theorem, Weierstrass's theorem on uniform convergence on compact sets, Bilinear transformations, Multivalued Analytic Functions, Riemann Surfaces.

3. **Advanced Analysis:** Elements of Metric Spaces, Convergence, continuity, compactness, Connectedness, Weierstrass's approximation Theorem. Completeness, Bare category theorem, Labesgue measure, Labesgue integral, Differentiation and integration

4. **Algebra:** Symmetric groups, alternating groups, Simple groups, Rings, Maximal ideals, Prime ideals, integral domains, Euclidean domains, principal ideal domains, Unique Factorisation domains, quotient fields, Finite fields,

Algebra of Linear Transformations, Reduction of matrices to Canonical Forms, Inner Product Spaces, Orthogonality, quadratic Forms, Reduction of quadratic forms.

5. **Numerical analysis:** Finite differences, interpolation ; Numerical solution of algebric equation; Iteration; Newton–Rephson method; Solution on linear system; Direct method; Gauss elimination method; Matrix–Inversion, elgenvalue problems; Numerical differentiation and integration. Numerical solution of ordinary differential equation; iteration method, Picard's method, Euler's method and improved Euler's method.

6. **Linear Programming Basic Concepts:** Convex sets. Linear Programming Problem (LPP). Examples of LPP, Hyperplane, open and closed half – spaces. Feasible, basic feasible and optimal solutions. Extreme point and graphical method. Simplex method, Duality in linear programming. Transformation and assignment problems.

7. **Measure Theory:** Measurable and measure spaces; Extension of measures, signed measures, Jordan – Hahn decomposition theorems. Integration, monotone convergence theorem, Fatou's lemma, dominated convergence theorem. Absolute continuity. Radon Nikodym theorem, Product measures, Fubini's theorem.

8. **Functional Analysis:** Banach Spaces, Hahn – Banach Theorem, Open mapping and closed Graph Theorems. Principle of Uniform boundedness, Boundedness and continuity of Linear Transformations. Dual Space, Embedding in the second dual, Hilbert Spaces, Projections. Orthonormal Basis, Riesz – representation theorem. Bessel's inequality, parsaval's identity, self adjoined operators, Normal Operators.

9. **Ordinary and partial differential equations:** Introduction to differential equations, Linear Equations with Variable Coefficients, Euler's Method, The Existence and Uniqueness Theorem, Homogeneous equations with constant coefficients, Fundamental solutions, linear independence, Wronskian, Non-homogeneous equations: method of undetermined coefficients, Non-homogeneous equations: method of variation of parameters, Partial differential equations, Monge's method, Canonical forms, Characterization of a partial differential equation, Heat equation, Wave equation, Laplace's equation

10. **Topology:** Topological spaces, open sets, closed sets, neighbourhoods, Inetrior, exterior and boundary of sets. Metric spaces, Induced Topology, Complete metric spaces, compactness in metric spaces, Continuity and homeomorphism, Connectedness, Compactness, Countability and separability.