

Kumaun University, Nainital

Minutes of meeting of the Board of Studies (BOS) in Mathematics

A meeting of the Board of Studies in CBCS Mathematics for M.A/M.Sc was held on 17/08/2020 at Kumaun University administrative block Nainital. The following members were present in the meeting:

1. Prof. Jaya Upreti, S. S. J. Campus, Almora (Convener & Head)
2. Prof. M. Hasan Shahid , Department of Mathematics , Faculty of Natural Sciences Jamia Millia Islamia , New Delhi-110025(External Expert) (Online)
3. Prof. S. B. Singh, Department of Mathematics , Computer science and Statistics, G. B. Pant Agriculture University, Pantnagar (External Expert) (Online)
4. Prof. V. P. Pande, S. S. J. Campus, Almora
5. Prof. M. C. Joshi, D. S. B. Campus, Nainital
6. Prof. B.C.Tewari, S. S. J. Campus, Almora (Online)

The committee resolves the followings:

1. The course-structure for M. A. / M. Sc. Mathematics on Choice Base Credit System (CBCS) of Department of Mathematics Kumaun University, Nainital was framed and approved.

Fri, Aug 7, 11:02
AM (7 days ago)

Prof. M. Hasan Shahid

to me

Dear Prof.Jaya Upreti,

Thank you for your mail and the attached syllabus for M.Sc. I have the following observations regarding the syllabus.

1. The syllabus of course 2, Abstract Algebra seems to be more advanced.
2. It is better to add some recent good books given below in (Core course 3), Differential Geometry and (Elective 15) Riemannian Geometry for better understanding of the subject. The exact title, author and publisher can be found in Google.

- (a) Elementary Differential Geometry, B.O.Niell. Academic Press
- (b) Curves and surfaces, Do Carmo
- (c) Elementary Differential Geometry, Andrew Pressley, SPRINGER
- (d) Riemannian Geometry, Do Carmo

Thanking you and with regards

Mohammad Hasan Shahid

drsurajbsingh@yahoo.com

Thu, Aug 13, 9:31
AM (1 day ago)

to me

Dear Prof. Jaya Upreti,

I have gone through the proposed syllabus for CBCS. It seems to be okay. I have mentioned credit against the subject, you may consider for future course of action.

Kind regards,

Dr. S. B. SINGH

Professor

Dept. of Maths, Stats. & Comp. Sc.,

G B Pant Univ. of Agriculture & Technology,

Pantnagar,

U S Nagar, Uttarakhand-263145, India

E-mail: drsurajbsingh@yahoo.com

On Tuesday, 4 August, 2020, 11:08:34 am IST, Jaya Upreti <prof.upreti@gmail.com> wrote:

KUMAUN UNIVERSITY

NAINITAL

Department of Mathematics

M.A./M.Sc Mathematics

Syllabus Under Choice Based Credit
System (CBCS)

2020

Semesters	Core Courses			Elective Course			Open Elective Course			Total Credits
	No. of Papers	Credits (L+T+P)	Total	No. of Papers	Credits (L+T+P)	Total	No. of Papers	Credits	Total	
I	05	15+5+0	20	-	-	-	-	-	-	20
II	05	15+5+0	20	-	-	-	-	-	-	20
III	02	(06+02+0)	08	2/3	6+2+0	08	1	(3+1+0)	04	20
IV	01+Viva-Voce	(03+01+0)+04	08	2/3	6+2+0	08	1	(3+1+0)	04	20
	13 core Courses		56			16		08		80

Kumaun University, Nainital

Department of Mathematics

M.A/M.Sc Mathematics

Syllabus under Choice Based Credit System (CBCS)

There shall be four semesters in the two-year M.A/M.Sc. programme in Mathematics for the Masters degree in Mathematics. There will be 20 credits in each semester. In the PG programme a theory course will not exceed four credits. A course having theory and tutorial both will be assigned maximum of four credits. Viva-voce will be assigned 4 credits. Offering of elective/open elective courses will depend on students interest and availability of teachers. There will be continuous assessment of the students throughout the semester. 75% of attendance is mandatory for students to appear in the semester end examination. There will be two mid-term evaluations for each course in each semester in the form of examination/assignments. One mid-term examination is compulsory for each course with a maximum of 30% of total marks. An evaluation of 20% of total marks for each course in each semester will be in the form of examination/assignments/performance of student in the classroom. The semester end examination for each course in each semester will be of 50% of total marks. Question Paper Structure for semester end examination: Duration of the semester-end examination will be two hours. Each paper in the external examination will be of 75 marks and will comprise of two sections : A and B. Section A will be of 30 marks and shall contain 8 questions out of which five questions will be attempted. Section B will be compulsory and of 45 marks with internal choice.

Core Courses:

S.N.	Code	Title	Lecture	Tutorial	Practical	Credits
1.		Real analysis	3	1	0	4
2.		Topology	3	1	0	4
3.		Differential Geometry	3	1	0	4
4.		Numerical Analysis	3	1	0	4
5.		Fluid Mechanics	3	1	0	4
6.		Complex Analysis	3	1	0	4
7.		Abstract Algebra	3	1	0	4
8.		Differential Equations	3	1	0	4
9.		Measure Theory	3	1	0	4
10.		Tensor Calculus	3	1	0	4
11.		Linear Algebra	3	1	0	4
12.		Dynamics of Rigid Bodies	3	1	0	4
13.		Functional Analysis	3	1	0	4
14.		Viva-Voce	0	0	0	4

Elective Courses for third Semester:

S.N.	Code	Title	Lecture	Tutorial	Practical	Credits
1.		Mathematical Statistics	3	1	0	4
2.		Number Theory	3	1	0	4
3.		Riemannian Geometry	3	1	0	4
4.		Special Functions	3	1	0	4
5.		Advanced Abstract Algebra	3	1	0	4
6.		Numerical Solution of ODE& PDE	3	1	0	4
7.		Relativity	3	1	0	4
8.		Introduction to Computer Programming with MATLAB	3	1	0	4

Elective Courses for Fourth Semester:

S.N.	Code	Title	Lecture	Tutorial	Practical	Credits
1.		Discrete Mathematics	3	1	<u>0</u>	4
2.		Calculus of Variations and Integral Equations	3	1	<u>0</u>	4
3.		Fourier Analysis	3	1	0	4
4.		Statistical Analysis	3	1	0	4
5.		Mathematical Computations and Computer Programming	3	1	0	4
6.		Dynamical Systems	3	1	0	4
7.		Operation Research	3	1	0	4
8.		Object Oriented Programming with C++	3	1	0	4

Open Elective Course for Third & Fourth Semester:

S.N	Code	Title	Lecture	Tutorial	Practical	Credits
1.		Business Mathematics	3	1	0	4
2.		Financial Mathematical	3	1	0	4

Semester Wise Course Structure

First Semester	Second Semester	Third Semester	Fourth Semester
Real Analysis	Complex Analysis	Linear Algebra	Functional Analysis
Topology	Abstract Algebra	Dynamics of Rigid Bodies	Elective1
Differential Geometry	Differential Equations	Elective1	Elective2
Numerical Analysis	Measure Theory	Elective2	Elective3(Open)
Fluid Mechanics	Tensor Calculus	Elective3(Open)	Viva Voce

Detailed Syllabus of Core Courses

Core Course 1: Real Analysis

Unit 1. Metric spaces: metric, Various examples, of metric spaces, open sets, interior of a set, Structure of open subsets of the real line, limit points, closed sets, closure of a set, Cauchy sequences, completeness.

Unit 2. Functions of several variables: Concept of functions of two variables, Simultaneous and iterated limits in functions of two variables, Partial derivatives: Definition, Existence and continuity, Interchange of order of differentiation, Directional derivatives.

Unit 3. Composite functions, Linear Continuity of function of two variables, differentiability of functions of two variables, Taylor's Theorem.

Unit 4. Linear transformation, Vector Valued functions, Differentiation of vector valued functions, inverse function theorem, implicit function theorem.

Books recommended:

1. *S. C. Malik and Savita Arora: Mathematical Analysis, New Age International.*
2. *G.F. Simmons: Introduction to Topology and Modern Analysis, Tata McGraw Hill.*
3. *W. Rudin: Principles of Mathematical Analysis (3rd edition), Tata Mc Graw Hill Kgakusha, International Student Edition, 1976.*
4. *T. M. Apostol: Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.*

Core Course 2: Topology

Unit 1. Basic concepts in Topology: Topology on a set, a topological space with examples, topologies on the real number system.

Unit 2. Neighborhood of a point/set, Open and closed sets, interior, boundary, closure, limit point, Derived sets of a set, Base and sub-base of a topology, Separable Spaces, First and Second Countable spaces,

Unit 3. Continuous map, open and closed maps, homeomorphisms, Topological invariants, Pasting Lemma, Subspaces, product spaces, quotient space.

Unit 4. Compactness, Compact spaces, Compactness of a metric space, Connectedness, connected space, path wise connected space, components. Separation axioms: $T_1, T_2, T_3, T_{3\frac{1}{2}}, T_4$, regular, completely regular and normal space.

Books Recommended:

1. *J. R. Munkres: Topology: Narosa Publishing House.*
2. *Shaum's outlines series: Tata McGraw Hill.*
3. *K. D. Joshi: Introduction to General Topology, Wiley Eastern, 1983.*
4. *G. F. Simmons: Introduction to Topology and Modern Analysis, McGraw Hill, 1963.*
5. *M. D. Raisinghania & R. S. Aggarwal: Topology, S. Chand & Co.*

Core Course 3: Differential Geometry

Unit 1. Curve in space, parameterized curves, regular curves, helices, arc length, reparametrization (by arc length), Tangent, principal normal, binormal, osculating plane, normal plane, rectifying plane, curvature torsion of smooth curves, Frenet- Serret formulae, Frenet approximation of space curve.

Unit2. Order of contact, osculating circle, osculating sphere, Spherical indicatrices, involutes and evolutes, Bertrand Curves, intrinsic equations of space curves, isometries of R^3 , Fundamental theorem of space curves, surfaces in R^3 .

Unit3. Regular Surfaces, coordinates neighborhoods, parameterized surfaces, change of parameters, level sets of smooth functions on R^3 , surfaces of revolution, mean curvature, tangent vector, first and second fundamental forms, classification of points on a surface

Unit4. Curvature of curve on surfaces, normal curvature, Meusnier theorem, principle curvatures, geometric interpretation of principal curvatures, Euler theorem, mean curvature, line of curvature, Rodrigue's formula, umbilical points, minimal surfaces, definition and examples, Gaussian curvature, intrinsic formulae, for the Gaussian curvature isometries of surfaces, Gauss Theorem Egregium (statement only).

Books Recommended:

1. *C.E. Weatherburn: Riemannian Geometry and Tensor Calculus.*
2. *Andrew Pressley: Elementary Differential Geometry, Springer (Undergraduate Mathematics Series), 2001.*
3. *J. A. Thorpe: Elementary Topics in Differential Geometry, Springer (Undergraduate Texts in Mathematics), 1979.*
4. *D. Somasundaram: Differential Geometry, A First Course, Narosa Publishing House, New Delhi, 2005.*
5. *T.J. Willmore: An Introduction To Differential Geometry, Oxford University Press.*
6. *R.S. Mishra: A Course in tensors with applications to Riemannian Geometry, Pothishala Pvt. Ltd. Allahabad, 1965.*
7. *Elementary Differential Geometry, B.O.Niell. Academic Press.*
8. *Curves and surfaces, Do Carmo.*
9. *Elementary Differential Geometry, Andrew Pressley, Springer.*
10. *Riemannian Geometry, Do Carmo.*

Core Course 4: Numerical Analysis

Unit 1. Errors in numerical Calculations: Absolute, Relative, Percentage errors, General Error, Error in series approximation. Solutions of Algebraic and Transcendental Equation: Bisection method, false position method, Newton-Raphson and generalized Newton's Method, Graffe's root squaring method, Lin Bairstow's method, Picards iteration method, convergence and error estimates of iterative methods.

Unit 2. Linear systems of equations: Consistency of Linear System of equations, Solutions of Linear Systems by direct method: Gaussian elimination and computation of inverse of a matrix, Method of Factorization, Solutions of Tridiagonal systems and ill conditioned linear systems. Solutions of linear systems by iterative methods: Jacobi method, Gauss- Siedel method.

Unit 3. Interpolation and curve fitting: Errors in Polynomial interpolation, Finite differences, Differences of a polynomial, Newton's forward and backward interpolation, Central differences, Gauss, Stirling, Bessel's and Everett's Formulae, Practical interpolation and interpolation with unevenly spaced points, Lagrange's Interpolation formula, Divided difference and Newton's General interpolation formula, Least square curve fitting procedure.

Unit 4. Numerical differentiation and integration: Numerical differentiation, cubic Spline method, Maximum and minimum values of tabulated function, Newton-Cotes Integration formula, Numerical integration by Trapezoidal rule, Simpson's 1/3, Simpson's 3/8, Weddle's rule and Romberg Integration, Numerical solution of ODE by Picard's Euler's Modified Euler's and Runge-Kutta methods.

Practical assignments: Based on topics included in the paper with (preferably with MATLAB).

Books Recommended:

1. S. S. Sastry: *Introductory Methods Numerical Analysis*, Prentice- Hall of India.
2. C.F. Gerald and P. O. Wheatley: *Applied Numerical Analysis*, Addison- Wesley, 1998.

Core Course 5: Fluid Mechanics

Unit 1. Lagrangian and Eulerian methods, Equation of continuity, Boundary surface, Stream lines, Velocity potential, Euler's equation of motions, Bernoulli's theorem, Helmholtz equations, Cauchy's integral, Equation of action under impulsive forces, Principle of energy.

Unit 2. Motion in two dimensions, Velocity potential and current functions, Sources and sinks, Doublet and images, Circle theorem, Motion of circular and elliptic cylinder in two dimensions, Joukowski transformation, Motion in three dimensions, Three dimensional sources, Sinks and doublets, Image of source in front of sphere, Motion of spheres, Stroke's stream function.

Unit 3. General theory of irrotational motion, Permanence of irrotational motion circulation, Stroke's theorem, Kelvin's circulation theorem, Green's theorem, Kelvin's minimum energy theorem, Conformal Representation, Kutta and Joukowski transformation, Theorems of Schwartz Christoffel.

Unit 4. Vortex motion: Rectilinear vortices, Rectilinear vortex with a circular section, An infinite row of parallel rectilinear vortices, Karman stream, Use of conformal transformation, Vortex pairs.

Books Recommended:

1. A. S. Ramsey: *A Treatise on Hydrodynamics*.
2. W. H. Besant and A. S. Ramsey: *A Treatise on Hydrodynamics*, CBS Publisher and Distributors, Delhi, 1988.

3. *F. Chorlton: A Text Book of Fluid Dynamics, CBC, 1985.*
4. *S.W. Yuan: Foundations of Fluid Dynamics, Prentice-Hall of India, 1988.*
5. *M. D. Raisinghania: Fluid Dynamics, S. Chand, 1939*

Core Course 6: Complex Analysis

Unit 1. Conformal mappings, Power series representation of analytic functions, Analytic functions as mappings, Riemann sphere, Linear transformations, Mobius transformation, Cross ratios, Mobius transformation on circles.

Unit 2. Derivative of an analytic function, Higher order derivatives, Cauchy's theorem integral formula. Morera's theorem, Cauchy inequality and Liouville's theorem.

Unit 3. Counting zeros, The open mapping theorem, Maximum modulus principle, Schwarz lemma, The fundamental theorem of algebra.

Unit 4. Entire functions, Hadamard's three circle theorem, Jensen's formula, Meromorphic functions.

Books Recommended:

1. *L.V. Ahlforse: Complex Analysis, Tata McGraw Hill.*
2. *J.B. Conway: Functions of one Complex variable, Springer-Verlag, 1980.*
3. *D. Sarason: Complex Function Theory, Hindustan Book Agency, Delhi, 1994.*
4. *B. Choudhary: Elements of Complex Analysis, Wiley Eastern Ltd., New Delhi, 1993*

Core Course 7: Abstract Algebra

Unit 1. Normal and subnormal series, composition series, Jordan Holder theorem, chain conditions.

Unit 2. Commentators. Solvable groups, solvability of subgroups and factor groups. Nilpotent groups and their equivalent characterizations.

Unit 3. Rings, ideals, prime and maximal ideals, quotient rings. Factorisation theory in commutative domains. Prime and irreducible elements, Euclidean Domains. Principal Ideal Domain. Divisor chain condition. Unique Factorisation Domains, examples and counter examples. Polynomial rings over domains. Eisenstein's irreducibility criterion. Unique factorisation in polynomial rings over U.F.D.s.

Unit 5. Fields, finite fields, field extensions, Galois extensions.

Books Recommended:

1. *J. Gallian: Abstract Algebra, Narosa Publication.*
2. *N. Jacobson: Basic Algebra, Vol. I, Hindustan Publishing Co., New Delhi.*
3. *M. Artin: Algebra, Prentice Hall of India.*
4. *Ramji Lal: Fundamentals in Abstract Algebra, Chakra Prakashan, Allahabad, 1985.*
5. *I. N. Herstein: Topics in Algebra, Wiley Eastern Ltd., N.D., 1975.*
6. *D. S. Dummit and R. M. Foote: Abstract Algebra, John Wiley, N. Y.*

Core Course 8: Differential Equations

Unit 1. Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs., General theory of homogeneous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Unit 2. Formation of P.D.E.'s. First order P.D.E.'s, Classification of first order, P.D.E.'s, Complete, general and singular integrals, Lagrange's or quasi-linear equations, Integral surfaces through a given curve. Orthogonal surfaces to a given system of surfaces, Characteristic curves.

Unit 3. Pfaffian differential equations, Compatible systems, Char pit's method, Jacobi's Method. Cauchy problem for first order PDEs.

Unit 4. Linear equations with constant coefficients, Reduction to canonical forms, Classification of second order P.D.E.s. General solution of higher order PDEs with constant coefficients.

Books Recommended:

1. M. D. Raisinghania, *Advanced Differential Equations*, S. Chand, 2016.
2. D.P. Choudhary and H. I. Freedman: *A Course in Ordinary Differential Equations*, Narosa Publishing House, New Delhi, 2002.
3. E.A. Coddington: *AN Introduction to Ordinary Differential Equations*, Prentice Hall of India, New Delhi, 1968.
4. T. Amaranath: *An Elementary Course in Partial Differential Equations*, Narosa Publishing House, New Delhi, 2005.
5. Erwin Kreyszig: *Advanced Engineering Mathematics*, John Wiley &SON Inc., New York, 1999.

Core Course 9: Measure Theory

Unit1. Countable sets, uncountable sets, relation between the cardinality of a nonempty set and the cardinality of its power set; Boolean ring, σ -ring, Boolean algebra and σ -algebra of sets, Set function.

Unit2. Introduction, Outer measure, Measurable sets and Lebesgue measure, Example of nonmeasurable sets, Measurable functions.

Unit3. The Riemann integral, The Lebesgue integral of a bounded function over a set of finite measure, The integral of nonnegative functions. The general Lebesgue integral, Convergence in measure.

Unit 4. Differentiation of monotone functions, Functions of bounded variation, Differentiation of an integral, Absolute continuity, Convex functions.

Books Recommended:

1. P. K. Jain: *Measure Theory*, New Age International.
2. P. R. Halmos: *Measure Theory*, Grand Text Mathematics, 14 Springer, 1994.
3. E. Hewit and K. Stromberg: *Real and Abstract Analysis*, Springer, 1975.

4. K.R. Parthasarathy: *Introduction to Probability and Measure*, TRIM 33, Hindustan Book Agency, New Delhi, 2005.

5. I. K. Rana: *An Introduction to Measure and Integration*, (Second Edition), Narosa Publishing House, New Delhi, 2005.

Core Course 10: Tensor Calculus

Unit 1. n-dimensional real vector space, Contravariant vectors, Dual space, Covariant vectors, tensor product, second order tensor, tensors of type (r,s). symmetric and skew symmetric properties, conjugate symmetric tensors, fundamental algebraic operations,: Addition, multiplication, contraction and inner product, Quotient law of tensor.

Unit 2. Metric tensor, length of a curve, magnitude of vector, Angle between two vectors, associated tensors.

Unit 3. Christoffel symbols, transformation rule and group property, covariant derivative, intrinsic derivative, Gradient, divergence curl.

Unit 4. An introduction to Riemann- Christoffel tensor, covariant Riemann- Christoffel tensor, Ricci tensor, scalar curvature and Bianchi identity.

Books Recommended:

1. R.S. Mishra: *A Course in tensors with applications to Riemannian Geometry*, Pothishala Pvt. Ltd., Allahabad, 1965.

2. K. Yano: *The theory of Lie derivatives and its applications*, North-Holland Publishing Company, Amsterdam, 1957.

3. Matthew S. Smith: *Principal and Application of Tensor Analysis*, W. Sons (Indianapolis) 1963.

4. N. J. Hicks: *Notes on differential geometry*, Van Nostrand publishing.

5. H.S. Shukla, Prasad & Dhruwa Narain Dubey: *Differential Geometry of Manifolds*, Vandana Prakashan, Mohanlalpur, Gorakhpur.

Core Course 11: Linear Algebra

Unit 1. A brief review of vector space, Inner products, Orthogonality, Best approximations, Projections, Cauchy-Schwartz inequality.

Unit 2. Adjoint of a linear transformation, Self adjoint transformations, Unitary operators.

Unit 3. Normal operators: Definition and properties and Spectral theorem.

Unit 4. Eigen vectors and eigen values of a linear operator, Minimal polynomial of a linear operator and its relations to characteristic polynomial, Caley-Hamilton theorem.

Books Recommended:

1. Hadley: *Linear Algebra*.

2. Hoffman and Kunz: *Linear Algebra*, Prentice Hall of India, New Delhi, 1972.

3. H. Helson: *Linear Algebra*, Hindustan Book Agency, New Delhi, 1994.

4. K. B. Dutta: *Matrix and Linear Algebra*, Prentice Hall of India.

Core Course 12: Dynamics of rigid Bodies

Unit 1. D'Alembert's principle, Motion about a fixed axis (Finite and Impulsive forces).

Unit 2. Motion in two dimensions under Finite and Impulsive forces, Principle of conservation of momentum and energy.

Unit 3. Lagrange's equations in generalized co-ordinates.

Unit 4. Hamilton's principle, principle of least action, Euler's geometrical and dynamical equations.

Books Recommended:

1. *S. L. Loney: Dynamics of rigid bodies.*

2. *Bhu Dev Sharma: Dynamics of Rigid Bodies, Kedarnath Ramnath Sons, 1984.*

3. *M. Ray & Harswarup Sharma: A text book of Dynamics of Rigid Body, Students' Friends & Co., Agra-2, 1971.*

4. *A. S. Ramsey: Dynamics – Part II.*

5. *H. Goldstein: Classical Mechanics, Narosa, 1990.*

Core Course 13: Functional Analysis

Unit 1. Metric convergence of sequences, Normed spaces, Banach Space, Properties of Normed spaces, Finite dimensional normed spaces and subspaces; Compactness and finite dimension, linear operators, Bounded and continuous linear operators; Linear functional; linear operators and functional on finite dimensional spaces, Normed spaces of operators, Dual space.

Unit 2. Inner product space; Hilbert space; Properties of Inner product spaces, Orthogonal complements and direct sums, Orthonormal sets and sequences; Hilbert adjoint operators, Self-Adjoint, Unitary and normal operators.

Unit 3. Fundamental Theorems of Normed and Banach Space: Zorn's Lemma, Hahn Banach Theorem, Hahn Banach Theorem for complex vector spaces and normed spaces, Applications to bounded linear functionals on $C[a, b]$, Adjoint operators, Uniform boundedness theorem, strong and weak convergence, convergence of sequences of operators and functional, Applications of summability of sequences, Open mapping theorem and closed graph theorem.

Unit 4. Banach contraction principle, Applications of Banach's theorem to linear, differential and integral equations, Approximation in Normed spaces, Uniqueness, strict convexity, Uniform approximation, approximation in Hilbert spaces.

Books Recommended:

1. *Erwin Kreyszig: Introductory Functional Analysis, Wiley India edition.*

2. *G. F. Simmons: Introduction to Topology and Modern Analysis, McGraw Hill, 1963.*

3. *A. E. Taylor: Introduction to Functional Analysis, John Wiley, 1958.*
4. *R. E. Edwards: Functional Analysis, Holt Rinehart and Winston, 1965.*

Core Course 14: Viva-Voce

1. In this paper evaluation will be based on the student's performance in viva voce test on all courses studied in the PG Programme.
2. During the viva-voce examination subject knowledge of the students based on the courses studied during the course program will be tested by the examiners.
3. There shall be one external and one internal examiner to conduct the Viva-voce examination.

Elective Courses for Third Semester

Elective 1: Mathematical Statistics

Unit 1. Descriptive Statistics: Measures of central tendency, dispersion skewness and kurtosis
Elements of probability: Sample space, discrete probability, independent events, Baye's theorem, random variables and distribution functions (univariate, bivariate, and generalization to multivariate).

Unit 2. Mathematical expectation and moments: Moment generating function, Characteristic function and cumulants. Probabilistic inequalities (Tchebychev, Markov and Jensen). Modes of convergence: weak and strong laws of large numbers. Central limit theorem (i.i.d. case). Markov chains with finite and countable state space, Poisson and birth- and- death processes.

Unit 3. Some standard discrete and continuous univariate distributions (Binomial, Poisson, Normal, Gamma and Beta), Distribution of order statistics and range.

Unit 4. Correlation, Rank correlation. Regression lines. Multiple and partial correlation of three variables only, Data reduction techniques: Principal component analysis, discriminant analysis, cluster analysis, canonical correlation.

Books Recommended:

1. *M.G.Kendall: Advanced theory of statistics Vol. I &II, Charle's Griffin & Co.*
2. *R. Hogg and A Craig: Introduction to Mathematical Statistics, Mac Millan & Co.*
3. *C.E. Weatherbun: A first course in Mathematical Statistics, The English Language Book Society And Cambridge University Press,*

1961.

4. S.C. Gupta & V.K. Kapoor: *Fundamentals of Mathematical Statistics*, Sultan Chand & Co.

Elective 2: Number Theory

Unit 1. Prime Numbers, Unique Factorization theorem, Farey series, Irrational numbers, Congruences, Residues, Quadratic Reciprocity Law, Primitive roots.

Unit 2. Fermet's theorem, Wilson's theorem, Continued fractions, Approximation of irrational of rationals, Hurwitz theorem.

Unit 3. The fundamental theorem of arithmetic in $K(1), K(i), K(\rho)$, Diophantine equation $X^2 + Y^2 = Z^2$, $X^4 + Y^4 = Z^4$, $ax^2 + by^2 + cz^2 = 0$, Quadratic fields, The arithmetic functions: $d(n)$, $\sigma(n)$, $\mu(n)$ and $\varphi(n)$ including elementary result on their order and average order.

Unit 4. Representation of numbers by two and four squares- waring's probem, Elementry results on $g(k)$ and $G(k)$, The prime numbers theorem.

Books Recommended:

1. G. H. Hardy and E. M. Wright: *Introduction to the theory of numbers*, Oxford University Press, 4th Edition.

2. D. M. Burton: *Elementary Number Theory*, 6th Edition, Tata McGraw Hill.

3. Thomas Koshy: *Elementary Number Theory with Applications*, Academic Press, 2nd Edition.

4. Kenneth H. Rosen: *Elementary Number Theory and its Applications*, Addison-Wesley Publishing Company, 1986.

Elective 3: Riemannian Geometry

Unit 1. N-dimensional real vector space, Covariant vectors, Dual space, Contravariant vectors, tensor product, Other tensors of second order, Tensors of type (r,s). Algebraic Operations on tensors: Symmetric and skew symmetric properties, Fundamental algebraic operations, Inner product of vectors, Euclidean vector space.

Unit 2. Differentiable manifold, Lie-bracket, Tangent space, Connexions, Covariant derivatives, Curvature tensor, Parallelism. Lie derivative, Exterior derivative, Cartan's structural equations.

Unit 3. Riemannian geometry : Riemannian metric, Christoffel symbols, Curvature tensor with respect to Christoffel symbols, Differential operators, Geodesics, Geodesic coordinates, Riemannian curvature, Conformal curvature tensor, Frenet's formulae.

Unit 4. Sub-manifolds and Hypersurfaces : Normals, Gauss's formulae, Weingarten equations, Coordinate viewpoint, Lines of curvature, Generalized Gauss and Mainardi-Codazzi equations.

Books Recommended:

1. R.S. Mishra: *A Course in tensors with applications to Riemannian Geometry*, Pothishala Pvt. Ltd., Allahabad, 1965.

2. K. Yano: *The theory of Lie derivatives and its applications*, North-Holland Publishing Company, Amsterdam, 1957.

3. Matthew S. Smith: *Principal and Application of Tensor Analysis, W. Sons (Indianapolis) 1963.*
4. N. J. Hicks: *Notes on differential geometry, Van Nostrand publishing.*
5. H.S. Shukla, Prasad & Dhruwa Narain Dubey: *Differential Geometry of Manifolds, Vandana Prakashan, Mohanlalpur, Gorakhpur.*

Elective 4: Special Functions

Unit 1. Preliminaries, Gamma function and related functions, Gauss multiplication theorem, the hypergeometric differential equation, Gauss hypergeometric function.

Unit2. Integral representation of hypergeometric function, Evaluation of hypergeometric function, the confluent hypergeometric differential equation, Confluent hypergeometric function.

Unit 3. Bessel's equation, solution of Bessel's equation, Bessel's functions $J_n(x)$, Recurrence Formulae, Equations reducible to Bessel's equation, orthogonality of Bessel's Functions, A generating function for $J_n(x)$, Basic properties.

Unit 4. Legendre's equation, Legendre's polynomial $P_n(x)$, Legendre's function of the second kind $Q_n(x)$, General solution of Legendre's equation, Rodrigue's formula, Legendre polynomials, A generating function of Legendre's polynomial, Orthogonality of Legendre polynomials, Recurrence formulae for $P_n(x)$.

Books Recommended:

1. E.D. Rainville: *Special functions.*
2. Nirvikar Saran: *Special Functions.*
3. W.W. Bell: *Special Function for Scientists and Engineers, Dever publications, 2002,*
4. U.P. Singh: *Special Function & Their Application, WISDOM PRESS, 2012.*

Elective 5: Advanced Abstract Algebra

Unit 1. Modules over a ring. Endomorphism ring of an abelian group. R – Module structure on an abelian group M as a ring homomorphism from R to $\text{End}_Z \{M\}$. Submodules. Direct summands. Annihilators. Faithful modules. Homomorphism. Factor modules. Correspondence theorem, Isomorphism theorems.

Unit 2. $\text{Hom}_R[M, N]$ as an abelian group and $\text{Hom}_R[M, M]$ as a ring. Exact sequences. Five lemma. Products, coproducts and their universal property. External and internal direct sums.

Unit 3. Free modules. Homomorphism extension property. Equivalent characterisation as a direct sum of copies of the underlying ring. Split exact sequences and their characterisations. Projective modules. Injective modules. Divisible groups. Examples of injective modules. Boolean Algebra.

Unit 4. Factorisation of polynomials in extension fields. Splitting fields and their uniqueness. Seperable field extensions. Perfect fields. Seperability over fields of prime characteristic. Transitivity and seperability. Automorphism of fields. Dedekind's theorem. Fixed fields. Normal extensions. Splitting fields and normality. Normal closures.

Books Recommended:

1. J. A. Gallian, *Contemporary Abstract Algebra, Narosa Publication, 7th Edition.*

2. Vivek Sahai, and Vikas Bisht: *Algebra*, Narosa Publishing House 1999.
3. I. N. Herstein: *Topics in Algebra*, Wiley Eastern, 1963.

Elective 6: Numerical solution of ODE &PDE

Unit 1. Numerical Solution of ordinary Differential equations: Numerical solution of ODE by Picard's, Euler's, Modified Euler's and Runge-Kutta methods, Boundary value problems: Finite difference method, Shooting method.

Unit 2. Numerical Solution of Partial Differential equations: Classification of second order general PDE, Difference method.

Unit 3. Difference methods for Parabolic PDE. Heat conduction equation and its numerical solutions with finite difference methods (Two and three level difference methods).

Unit 4. Difference methods for Hyperbolic PDE. Wave equation and its numerical solutions with finite difference methods (First order only).

Unit 5. Difference methods for Elliptical PDE. Dirichlet problem for Laplace equation and its numerical solutions with finite difference methods.

Practical assignments: Based on topics included in the paper with Matlab/OCTAVE, Calculator is allowed in the Examination.

Books Recommended:

1. S. S. Sastry: *Introductory Methods Numerical Analysis*, Prentice- Hall of India.
2. M. K. Jain, S R K Iyengar, R K Jain: *Computational Methods for Partial Differential equations: New Age international*, New Delhi, 2016.
3. M. K. Jain: *Numerical Solutions of Differential Equations: New Age international*, New Delhi, 2014.
4. C. F. Gerald and P. O. Wheatley: *Applied Numerical Analysis*, Addison- Wesley, 1998.

Elective 7: Relativity

Unit 1. Special Relativity: Inertial Frames of reference, Michelson-Morley experiment, Doppler effect, Stellar aberration, Simultaneity, Postulates of special relativity, Lorentz transformation, Length contraction, Time dilation, Clock paradox, Addition of velocities and accelerations, Four-dimensional space time, Light cone, Mass variation, Velocity four vector, Momentum and force, Mass-Energy relationship.

Unit 2. General Relativity: Geodesics, Geodesic coordinates, Curvature tensor and its algebraic properties, Bianchi's identities, Contracted curvature tensor, Conditions for a flat space time, Displacement of space -time, Killing equations, Groups of motion, Space -time of constant curvature.

Unit 3. Principal of covariance, Non-inertial frames of reference, Principal of equivalence, Weak field approximation of geodesic equations, Law of gravitation in empty space-time, Canonical coordinates, Schwarzschild solutions.

Unit 4. Experimental tests of general relativity, Schwarzschild metric in isotropic coordinates, Brikhoff's theorem, Law of gravitation in non-empty space time.

Books Recommended:

1. D.F. Lawden: *An Introduction to tensor calculus and relativity*,

2. *J.V. Narlikar: General relativity and cosmology.*
3. *R.H. Good: Basic concept of relativity, 1978.*
4. *A.S. Eddington: Mathematical theory of relativity, 1981.*

Elective 8: Introduction to Computer Programming with MATLAB

Unit1. Introduction to programming: Components of a computer, Working with numbers Machine code, Software hierarchy.

Unit2. Programming Environment: MATLAB Window, A first program, Expressions, Constants, Variables and assignment statement, Arrays.

Unit3. Graph Plots: Basic plotting, Built in functions, Generating waveforms, Sound reply, load and save.

Unit4. Procedures and Functions: Arguments and return values, M-files, Formatted console input output, String handling, Conditional statements: If, Else, Else if, Repetition statements: While, For.

Books Recommended:

1.C.B.Molar, Numerical computing with MATLAB, SIAM, 2004, Available on- line at <http://www.mathworks.com/index ncm.html>.

Elective Courses for Fourth Semester

Elective 1: Discrete Mathematics

Unit 1. Principle of mathematical induction, partially ordered sets, Lattices: Lattices as partially ordered sets, Their Properties, Lattices and algebraic systems. Principle of duality, Sub lattices, Complete, Complemented and Distributive lattices.

Unit 2. Boolean algebra, Boolean functions, Boolean expressions, Applications to switching circuits.

Unit 3. Elements of graph theory: Basic terminology, Paths and circuits, Eulerian and Hamiltonian graphs, planar graphs, Directed graphs.

Unit 4. Trees: Rooted trees, path lengths, spanning trees, minimum spanning trees.

Books Recommended:

1. *C. L. Liu: Elements of discrete mathematics, Tata McGraw Hill Education, 2008.*
2. *Ram Babu: Disrete Mathematics, Pearson Edition India, 2011.*
3. *Lipschutz: Disrete Mathematics, Tata McGraw Hill, 2011.*

Elective 2: Calculus of Variations and Integral equations

Unit 1. Functionals and extremals, Necessary and sufficient conditions for extrema, Variation and its properties.

Unit 2. Euler equations, Cases of several dependent and independent variables, Variational methods for boundary value problems in ordinary and partial differential equations, Functionals dependent on higher derivatives, Parametric forms, Simple applications.

Unit 3. Classification of linear integral equations, Relation between differential and integral equations.

Unit 4. Method of successive approximation for Fredholm and Volterra equations, Resolvent kernel.

Books Recommended:

1. *L. Elsgolts: Differential Equations and Calculus of Variations, Mir Publishers, 1970.*
2. *A. S. Gupta: Calculus of Variations, Prentice Hall of India, New Delhi, 1999.*
3. *J. H. Davis: Methods of Applied Mathematics with a MATLAB Overview, Birkhäuser, Inc., Boston, MA, 2004.*
4. *L.G. Chambers: Integral Equations A short Course, Int. Text Book company Ltd. 1976 .*
5. *Abdul J Jerry: Introduction to Integral Equations with Applications, Marshal and Dekkar.*
6. *Naveen Kumar: An Elementary Course on Variational Problems in Calculus, Narosa, 2004.*

Elective 3: Fourier Analysis

Unit 1. Genesis and Basic Properties of Fourier Series: Derivation and Solutions of wave and heat equations, Definition of Fourier series and Examples, Uniqueness, convolution, Cesaro means and summation, Fejers theorem, Abel mean and summation, Poisonkernel and Dirichlet's problem in the unit Disc.

Unit 2. Convergence and some applications of Fourier Series: Mean square convergence of Fourier series, pointwise convergence, Isoperimetric inequality, Weyle's equidistribution theorem, continuous but nowhere differentiable function, Heat equation on the circle.

Unit 3. Fourier Transforms: Elementary theory and definition of Fourier transforms, Schwartz space, Fourier transform on S , The Fourier inversion, Plancherel formula, Weierstrass approximation theorem, Application of Fourier transform to some partial differential equations.

Unit 4. Poison summation formula, Theta and Zeta functions, Heat and Poisson Kernels, Heisenberg uncertainty principle.

Books Recommended:

1. *Elias M. Stein & Rami Shakarchi: Fourier Analysis, An Introduction: Levant Books, Kolkata*
2. *Rajendra Bhatia: Fourier Analysis.*
3. *E. C. Titchmarsh: A Theory of Functions, Oxford University, Press, 1939.*
4. *A. Zygmund: Trigonometric series Vol. I, The University Press, Cambridge, 1959.*

Elective 4: Statistical Analysis

Unit 1. Statistical Inference: Concept of consistency, efficiency, sufficiency, unbiasedness, and completeness. Existence of best asymptotically, normal estimates under regulatory conditions.

Unit 2. Maximum likelihood and other methods of estimation. Properties of maximum likelihood estimates. Minimax and Baye's estimates. Interval estimation: Neyman's Approach. Best confidence intervals.

Unit 3. Testing of Hypothesis: Simple and composite hypothesis, critical region, two types of errors, level of significance and power of a test. Most powerful test and uniformly most powerful test.

Unit 4. Neyman and Pearson's lemma. Likelihood Ratio tests. Large sample test. Sampling distribution of mean and variates. Exact sampling distributions: t, F and Z distributions and tests of significance based on them. Chi square distribution and its applications.

Books Recommended:

1. J. Medhi: *Stochastic Processes*, Wiley Eastern Ltd.
2. H.C. Saxena & P.U. Surendran: *Statistical Inference*, S. Chand & Co.
3. H.C. Saxena: *Mathematical Statistics* .
4. S.C. Gupta & V.K. Kapoor: *Fundamentals of Mathematical Statistics*, Sultan Chand & Co.
5. S.K. Sinha: *Reliability & Life Testing*.

Elective 5: Mathematical Computations and Computer programming

Unit 1. Introduction to Programming in C: Introduction to Algorithms & Flowcharts Variables, constant, Keywords, signed and unsigned modifiers.

Unit 2. Expression and operators: Arithmetic, logical and relational operators, bitwise operators, incremental operators, assignment operators. Functioning of these operators.

Control flow: If-else, switch, while, do-while, for loops, continue, break statements, Nesting of control statements and loops.

Unit 3. Working with functions: Variable and functions, Argument passing to functions, type of functions, storage classes, scope rule, C standard libraries.

Unit 4. Data-types : Structure, Union, enumeration.

Practical assignments: Based on topics included in the paper.

Books recommended:

1. Ritchier & Kernighan: *The C programming language*, Prentice Hall of India.
2. V. Rajaraman: *Computer Programming in 'C'*, Prentice Hall of India.
3. E. Balaguruswami: *Programming in ANSI 'C'*, Tata McGraw Hill.

Elective 6: Dynamical Systems

Unit 1. Examples of dynamical systems, Preliminaries from calculus, elementary definitions, Hyperbolicity, An example from quadratic family, symbolic dynamics.

Unit 2. Topological conjugacy, Chaos, structural stability, Sarkovskii's theorem, The Schwarzian derivative, Bifurcation theory.

Unit 3. Preliminaries from complex analysis, The Riemann sphere, Steriographic projection, Examples from quadratic maps.

Unit 4. Critical points, Exceptional points, Fixed points and their classification, Julia and Fatou sets, symbolic dynamics.

Books Recommended:

1. R. L. Devaney: *An Introduction to Chaotic Dynamical Systems*, Addison- Wesley.
2. A. F. Beardon: *Iteration of Rational Functions*, Springer- Verlag.
3. C. G. Carlson and T. W. Gamelin: *Complex Dynamics*, Universitext, Springer.
4. R. A. Holmgren: *A first course in discrete dynamical systems*, Springer.

Elective 7: Operation Research

Unit 1. Basics of OR and LPP: Development of OR, Definition, characteristics, scope, objectives and limitations of OR, Formulation of LPP, Graphical Method to solve LPP, General LPP, Canonical and Standard forms, Properties of Solutions and Theory of Simplex method, Big M Method and Two phase simplex method, Degeneracy in LPP. Duality in LPP, Duality and simplex method, Dual simplex method, Revised simplex method and bounded variable problems.

Unit 2. Integer and Dynamic Programming: Pure and Mixed integer programming, Gomory all IPP method, Fractional cut method, Lp Formulation of TP, Transportation Table, Finding initial basic feasible solution, Test of optimality, Degeneracy, MODI method, Stepping Stone method, Solutions of Assignment problems, Hungarian method, Duality in assignment problem.

Unit 3. Sensitivity Analysis: Changes in Objective Function Coefficient, Changes in constants, Changes in coefficients of decision variables in constraints, Structural changes.

Unit 4. Network Analysis and Nonlinear Programming: Network flow problem, minimal spanning tree problem, shortest route problem, maximal flow problem, minimum cost flow problems, critical path analysis, PERT and CPM, Formulation of NLPP, general NLPP, constrained optimization with equality and inequality constraints.

Books Recommended:

1. H.A. Taha: *Operations Research, An Introduction*, Pearson.
2. Kanti Swarup, P K Gupta, Manmohan: *Operations Research*, Sultan Chand & Sons, New Delhi.
3. S.S. Rao: *Optimization Theory and Applications* Wiley Eastern.
4. F. S. Hiller and G. J. Lieberman: *Introduction to Operation Research (6th Edition)*, McGraw-Hill International Edition, 1995.

Elective 8: Object Oriented Programming with C++

Unit1. Introduction to Object oriented programming, Object oriented programming paradigm, basic concept of Object oriented programming,, advantages of Object oriented programming.

Unit2. Input and Output Statements, arithmetic operators, logical, Boolean, bitwise operators, type conversion, derived data types, symbolic Constants, const modifier, if else statement, for loop, do while loop, switch statements, break, continue and goto statements.

Unit3. Classes, data members and member functions, default arguments, call by reference and return by reference, class scope, constructors and destructor, friend and virtual function.

Unit4. Operator overloading, type conversion, dynamic memory allocation, polymorphism (function overloading), inheritance, string manipulation.

Books Recommended:

1. *E Balaguruswami:* Object Oriented programming with C++, TMH

2. *Robert Lafore:* Object Oriented programming with C++, SAMS

Open Elective Courses

Open Elective 1. Business Mathematics

Unit 1: Matrices and Determinants

- (a) Algebra of matrices. Inverse of a matrix, Matrix Operation – Business Application
- (b) Solution of system of linear equations (having unique solution and involving not more than three variables) using matrix inversion Method and Cremer’s Rule, The Leontief Input Output Model (Open Model Only).

Unit 2: Calculus I

- (a) Mathematical functions and their types- linear, quadratic, polynomial, exponential,
- (b) Logarithmic function Concepts of limit, and continuity of a function
- (c) Concept and rules of differentiation, Maxima and Minima involving second or higher order Derivatives.
- (d) Concept of Marginal Analysis, Concept of Elasticity, Applied Maximum and Minimum Problems including effect of Tax on Monopolist’s optimum price and quantity, Economic Order Quantity.

Unit 3: Calculus II

- (a) Partial Differentiation: Partial derivatives up to second order; Homogeneity of functions and Euler’s theorem; Total differentials; Differentiation of implicit functions with the help of total differentials
- (b) Maxima and Minima: Cases of two variables involving not more than one constraint including the use of the Lagrangian multiplier.
- (c) Integration: Standard forms. Methods of integration – by substitution, by parts, and by use of partial fractions; Definite integration; Finding areas in simple cases
- (d) Application of Integration to marginal analysis. Consumer’s and Producer’s Surplus, Rate of Sales and the Learning Curve

Unit 4: Mathematics of Finance

- (a) Rates of interest-nominal, effective– and their inter-relationships in different compounding situations.
- (b) Compounding and discounting of a sum using different types of rates.

- (c) Types of annuities, like ordinary, due, deferred, continuous, perpetual, and their future and present values using different types of rates of interest. Depreciation of Assets.
(*General annuities to be excluded*)

Unit 5: Linear Programming

- (a) Formulation of linear programming problem (LPP). Graphical solution to LPP. Cases of unique and multiple optimal solutions. Unbounded solutions, infeasibility, and redundant constraints.

Books Recommended:

1. Wikes, F.M., *Mathematics for Business, Finance and Economics*. Thomson Learning.
2. Thukral, J.K., *Mathematics for Business Studies*.
3. Vohra, N.D., *Quantitative Techniques in Management*. McGraw Hill Education.
4. Soni, R.S., *Business Mathematics*. Ane Books, New Delhi.
5. Singh J. K., *Business Mathematics*. Himalaya Publishing House.

Open Elective 2: Financial Mathematics

Unit1. Introduction to Discrete probability, probability spaces, Independence, Binomial probabilities, Random Variables, Conditional probabilities, Expectation, Variance, standard deviation, Covariance and correlation, Best linear predictor, Stochastic process, Normal , lognormal and Cauchy variants, Filtrations and Martingales. The central limit theorem.

Unit2. Arbitrage, Return and Interest, The time value of money, Bonds, Shares and Indices, Models and Assumption **Deterministic cash flow:** Net present value, Internal rate of Return, A comparison of IRR and NPV, Bonds: price and Yield, Clean and dirty price, price yield curves, Duration term structure of Interest Rates, Immunizations, convexity, Callable Bonds. **Random cash flow:** Random Returns, Portfolio Diagrams and Efficiency, Feasible Sets, Markowitz model, Capital Asset Pricing model, Diversification, CAPAM as a pricing Formula, Numerical Techniques.

Unit3. Forwards and Futures: Forwards and Futures, Forwards and Futures price, Value of Future contract, Method of Replicating Portfolios, Hedging with Futures, Currency Futures, Stock Index Futures.

Unit4. Lognormal model, Geometric Brownian Motion, Suitability of GBM for the stock prices, Binomial tree model. Options: Call options, Put options, Put call parity, Binomial Options Pricing model, Pricing American options, Factor influencing option Premiums, Option an Assets with Dividends, Dynamic Hedging, Risk-Neutral Valuation.

Unit5. Risk-Neutral Valuation, The Black Scholes Formula, Option on Futures, Option an Assets with Dividends, black Scholes and BOPM, Implied Volatility, Dynamic Hedging, The Greeks, Te black Scholes PDE, Speculating with Options. Value of Risk: Definition of VaR, Linear model, Quadratic Model, Monte Carlo Simulation, The Martingale.

Books Recommended:

1. Amber Habib: *The calculus of Finance*, Universities Press Hyderabad.
2. Steven Roman: *Introduction to mathematics of Finance (Chapter1-9)*, Springer International Editions.

